# TRINITY COLLEGE

**Semester One Examination, 2022**

**Question/Answer booklet**

## MATHEMATICS APPLICATIONS UNIT 3

### Section One: Calculator-free

**SOLUTIONS**

WA student number:   In figures

In words _____

_____

Your name _____

## Time allowed for this section
Reading time before commencing work:   five minutes
Working time:   fifty minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section
**To be provided by the supervisor**
This Question/Answer booklet
Formula sheet

**To be provided by the candidate**
Standard items:   pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:   nil

## Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 51 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.
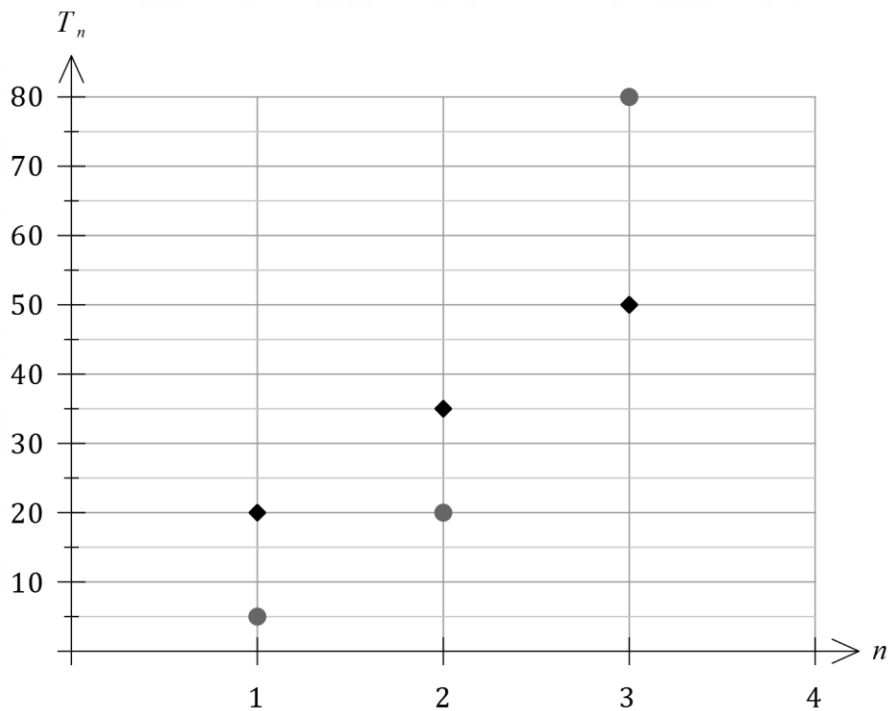
**Section One: Calculator-free** 35% (51 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

**Question 1** **(5 marks)**

The graph below shows the first few terms of an arithmetic sequence and a geometric sequence.



(a) Deduce a rule for the $n^{\text{th}}$ term of the geometric sequence. (2 marks)

| Solution |
|---|
| $T_n = 5(4)^{n-1}$ |
| |
| **Specific behaviours** |
| ✓ indicates correct common ratio $r$ |
| ✓ correct rule |

(b) Determine the $40^{\text{th}}$ term of the arithmetic sequence. (3 marks)

| Solution |
|---|
| $T_n = 20 + (n-1)(15)$ |
| $= 5 + 15n$ |
| $T_{40} = 5 + 15 \times 40$ |
| $= 5 + 600$ |
| $= 605$ |
| **Specific behaviours** |
| ✓ indicates correct common difference $d$ |
| ✓ formulates rule or expression for $T_{40}$ |
| ✓ correct value |

**Question 2** **(5 marks)**

The statements in parts (a) to (e) of this question relate to graph $G$ shown at right.

For each statement, state whether it is true or false and support your answer with brief reasons.
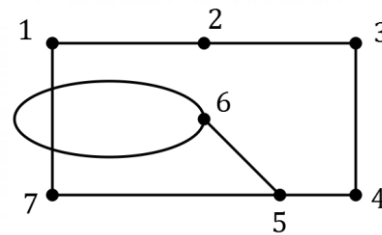


(a)     Graph $G$ has a bridge. (1 mark)

| Solution |
| --- |
| True - between vertices 5 and 6. |
| **Specific behaviours** |
| ✓ correct answer with justification |

(b)     Graph $G$ is planar. (1 mark)

| Solution |
| --- |
| True - it could be drawn in the plane so that no two edges cross. |
| **Specific behaviours** |
| ✓ correct answer with justification |

(c)     Graph $G$ has 7 even vertices. (1 mark)

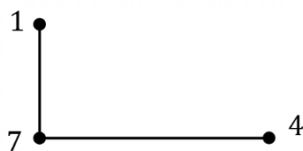| Solution |
| --- |
| False - only 5 vertices $(1, 2, 3, 4, 7)$ are even. |
| **Specific behaviours** |
| ✓ correct answer with justification |

(d)     Graph $G$ has a trail of length 9. (1 mark)

| Solution |
| --- |
| False - e.g., only 8 edges so maximum possible length of any trail is 8. |
| **Specific behaviours** |
| ✓ correct answer with justification |

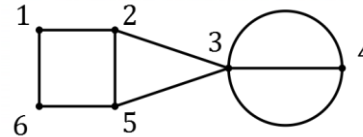(e)     Graph $F$, shown below, is a subgraph of graph $G$. (1 mark)



| Solution |
| --- |
| False - vertices 4 and 7 are adjacent in $F$ but not in $G$. |
| **Specific behaviours** |
| ✓ correct answer with justification |

**Question 3** **(7 marks)**

(a) Graph $G_1$ is shown at right.

Show that graph $G_1$ satisfies Euler's formula.



(2 marks)

| Solution |
|---|
| $v = 6, \quad f = 5, \quad e = 9$ |
| Hence $v + f - e = 6 + 5 - 9 = 2$, as required. |
| **Specific behaviours** |
| ✓ clearly indicates correct values for $v, f$ and $e$ |
| ✓ shows that $v + f - e = 2$ |

(b) A haulage company has three trucks and four drivers. The drivers that are licensed to drive each of the trucks are shown with a tick in the following table.

| | | Driver | | | |
|---|---|---|---|---|---|
| | | A | B | C | D |
| Truck | 1 | ✓ | | | ✓ |
| | 2 | ✓ | ✓ | ✓ | ✓ |
| | 3 | ✓ | ✓ | ✓ | |

(i) Represent the information in the table as bipartite graph $G_2$. (3 marks)

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ any graph that is clearly bipartite |
| ✓ all vertices marked with dot labelled |
| ✓ all edges correct |

(ii) Graph $G_2$ can be drawn in the plane. Determine, with justification, the number of faces $G_2$ has. (2 marks)

| Solution |
|---|
| Since $G_2$ is a connected planar graph, it will satisfy Euler's formula: |
| $$v + f - e = 2 \rightarrow 7 + f - 9 = 2 \rightarrow f = 4$$ |
| $G_2$ has 4 faces. |
| **Specific behaviours** |
| ✓ suitable reasoning or diagram |
| ✓ correct number of faces, with justification |

**Question 4**        **(10 marks)**

Over a period of one week, a group of children were observed and the total time that they slept each day recorded. The scatterplot below shows the age in years and months of each child on the $x$-axis and their average daily sleep time in hours on the $y$-axis.
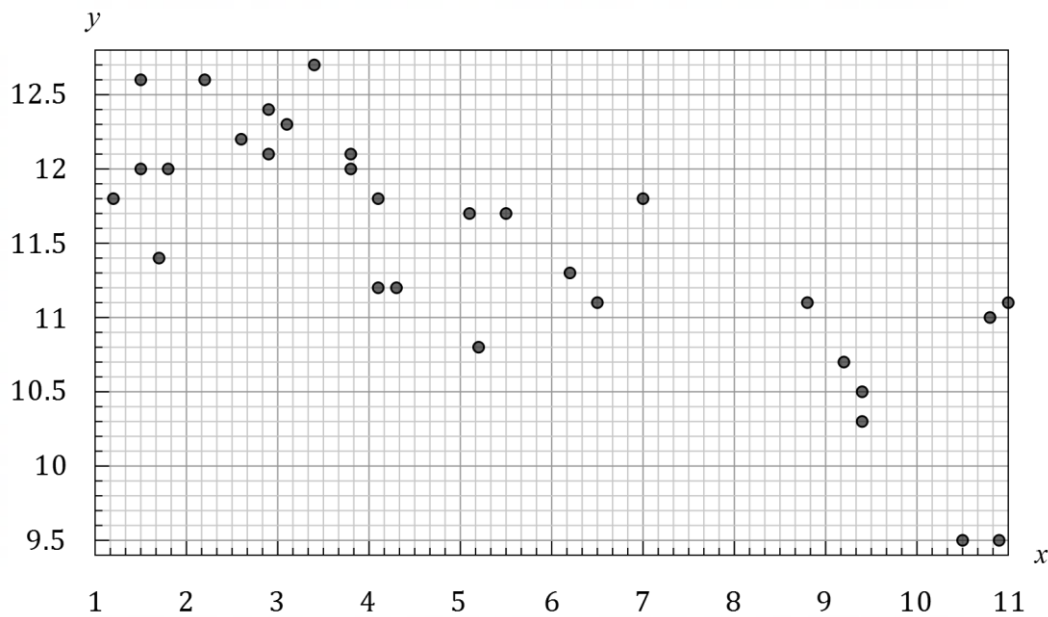


The equation of the least-squares line is $\hat{y} = 12.4 - 0.2x$ and $r_{xy} = -0.81$.

(a) How many children aged 2 were in the group?        (1 mark)

| Solution |
| --- |
| 4 children |
| **Specific behaviours** |
| ✓ correct number |

(b) Name the explanatory variable.        (1 mark)

| Solution |
| --- |
| **Age** of child. |
| **Specific behaviours** |
| ✓ correct variable |

(c) Describe the linear association between the variables in terms of direction and strength.

       (2 marks)

| Solution |
| --- |
| Association is negative and strong. |
| **Specific behaviours** |
| ✓ direction <br> ✓ strength |

(d)     Interpret the slope of the least-squares line in the context of this question.     (2 marks)

| **Solution** |
|---|
| For every extra year of age, the average daily sleep of a child is observed to decrease by 0.2 hours. |
| **Specific behaviours** |
| ✓ identifies as age increases sleep decreases |
| ✓ quantifies change using value of slope in context and units |

(e)     Predict the average daily sleep time of a child who has just had their eighth birthday.

(2 marks)

| **Solution** |
|---|
| $$\hat{y} = 12.4 - 0.2 \times 8$$ $$= 12.4 - 1.6$$ $$= 10.8 \, \text{h}$$ |
| **Specific behaviours** |
| ✓ substitutes into least-squares line |
| ✓ correct prediction |

(f)     A paediatrician looked at the data and said, " as children age, it causes them to need less sleep". Comment on this statement.     (2 marks)

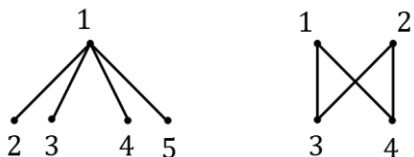| **Solution** |
|---|
| The observed association between the two variables is not a sufficient condition to establish a causal relationship and so the statement is not true. |
| **Specific behaviours** |
| ✓ takes issue with claim of a causal relationship |
| ✓ explains that observed association alone is not enough |

**Question 5** **(7 marks)**

A complete bipartite graph with $n$ vertices in one group and $m$ vertices in the other group will have $nm$ edges.

(a) Explain the meaning of the term **complete** for such a graph. (1 mark)

| Solution |
| --- |
| Complete: Every vertex of one group is connected to every vertex of the other group. |
| **Specific behaviours** |
| ✓ reasonable explanation |

(b) Draw a complete bipartite graph with 4 edges. The graph should clearly be bipartite with its vertices labelled $1, 2, 3, ...$ and so on. (2 marks)

| Solution |
| --- |



| **Specific behaviours** |
| --- |
| ✓ either bipartite graph shown, with required number of edges |
| ✓ labelled vertices |

(c) Construct the adjacency matrix for the graph you drew in part (b). (2 marks)

| Solution |
| --- |

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

| **Specific behaviours** |
| --- |
| ✓ square matrix (or table) filled with 0's and 1's. |
| ✓ correct matrix (or table) |

(d) Determine, with justification, the minimum number of vertices that a complete bipartite graph with 36 edges can have. (2 marks)

| Solution |
| --- |
| With 36 edges, the numbers of vertices in each set will be |
| $(1, 36) \to v = 37$ or $(2, 18) \to v = 20$ or $(3, 12) \to v = 15$ or $(4, 9) \to v = 13$ or $(6, 6) \to v = 12$ |
| Hence minimum number of vertices is 12. |
| **Specific behaviours** |
| ✓ correct minimum number |
| ✓ justifies answer by comparing to at least one other possibility |

**Question 6** **(8 marks)**

The graph below represents a town centre, with street corners shown as vertices, streets as edges and the length, in metres, of each street as an edge weight.
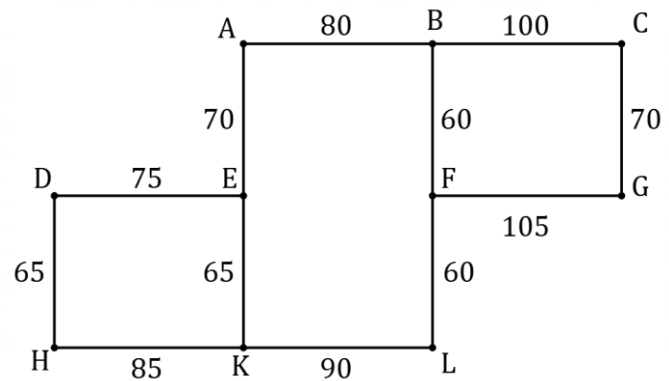
The total length of all streets in the town centre is 925 m.



(a) List, in order, the vertices that lie on the shortest path from $H$ to $C$ and state the length of this path in metres. (3 marks)

| Solution |
|---|
| $H - D - E - A - B - C = 390$ m |
| |
| **Specific behaviours** |
| ✓ any path with correct length |
| ✓ lists vertices on shortest path |
| ✓ states length of shortest path |

(b) Does the graph contain an Eulerian trail? Justify your answer. (2 marks)

| Solution |
|---|
| No - for an Eulerian trail to exist in a graph there must be no odd vertices, but this graph contains four. |
| |
| **Specific behaviours** |
| ✓ states no |
| ✓ states there must be no odd vertices for an Eulerian trail to exist |

(c) A cleaning team must make their way along every street in the town centre, starting and finishing at the same street corner. Determine the minimum distance they must walk and explain how they can achieve this minimum. (3 marks)
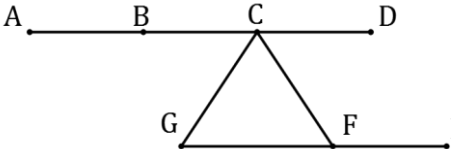
| Solution |
|---|
| To start and finish at the same vertex and travel along every edge once, graph must be Eulerian. To this end, add a duplicate edge $BF$ and a duplicate edge $EK$ (i.e., walk along these edges twice). Then the team can start and finish anywhere and walk an Eulerian trail. |
| |
| Hence the minimum distance is the sum of the weights plus the two extra edges: $d = 925 + 65 + 60 = 1050$ metres. |
| |
| **Specific behaviours** |
| ✓ identifies a possible walk that meets requirements |
| ✓ calculates a distance, supported by a possible walk |
| ✓ states correct minimum with reasons or lists a possible walk |

**Question 7**                                                     **(9 marks)**

A farm has seven buildings spread out over an area of land. There are paved roads between buildings A and B, B and C, C and D, C and F, C and G, E and F, and F and G.

(a)     Construct graph $G$ to represent this network of buildings and paved roads.     (2 marks)

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ correct number of labelled vertices |
| ✓ correct graph |

(b)     Determine the length of

       (i)     the longest trail in graph $G$.                                           (1 mark)

| Solution |
|---|
| Length is 6. (e.g., A-B-C-F-G-C-D) |
| **Specific behaviours** |
| ✓ correct length |

       (ii)     the shortest, closed walk in graph $G$ that visits all seven vertices.     (1 mark)

| Solution |
|---|
| Length is 11. (e.g., A-B-C-D-C-F-E-F-G-C-B-A) |
| **Specific behaviours** |
| ✓ correct length |

(c)     State, with justification, whether graph $G$ is a Hamiltonian graph.     (2 marks)

| Solution |
|---|
| No. The graph does not contain a Hamiltonian cycle. |
| **Specific behaviours** |
| ✓ states no |
| ✓ correct justification |

(d)     A new paved road is planned between two different buildings. With the addition of this road, graph $G$ will be a semi-Hamiltonian graph. Determine the number of possible pairs of buildings that the road could be built between. Justify your answer.     (3 marks)

| Solution |
|---|
| For a Hamiltonian path to exist, add an edge between any of the following pairs of vertices:<br><br>           A-D, A-E, A-G, D-B, D-E, D-G, E-B.<br><br>There are 7 possible pairs of buildings the road can run between. |
| **Specific behaviours** |
| ✓ lists one pair of vertices so that $G$ has a Hamiltonian path |
| ✓ lists at least 5 possible distinct pairs |
| ✓ states correct number of pairs |

**End of questions**                      

Supplementary page

Question number: _____